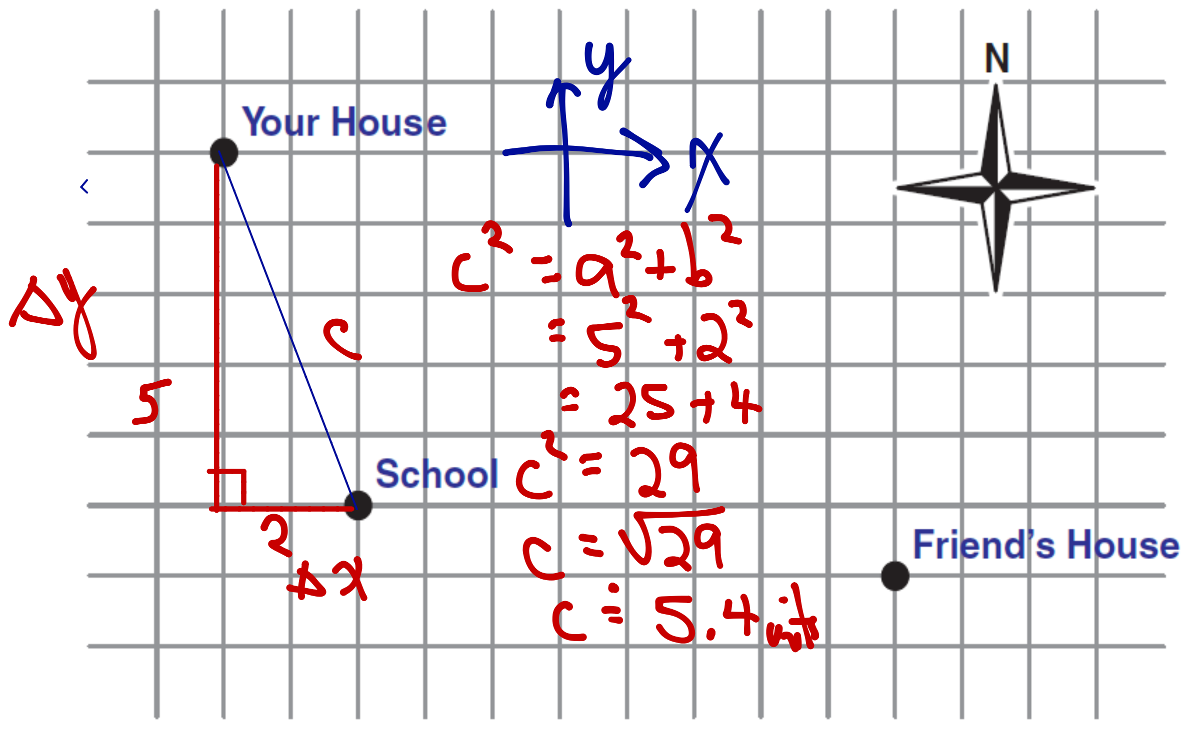
LENGTH OF A LINE SEGMENT

Learning Goal

- Determine the length of a line segment.

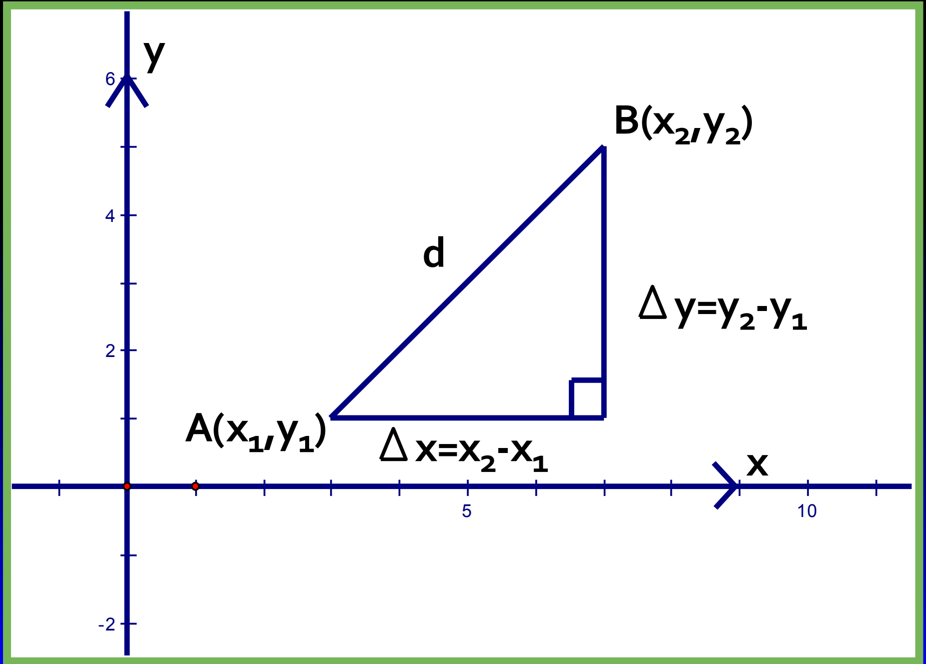
Minds On…

Suppose that the city in which you live has a system of evenly spaced perpendicular streets, forming square city blocks. The map below shows your school; your house, which is located two blocks west and five blocks north of the school; and your best friend's house, which is located eight blocks east and one block south of the school.



Big Ideas

- The distance, d, between the endpoints of a line segment, $A(x_1,y_1)$ and $B(x_2,y_2)$ can be calculated using the distance formula:



$$d^2 = \Delta x^2 + \Delta y^2$$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

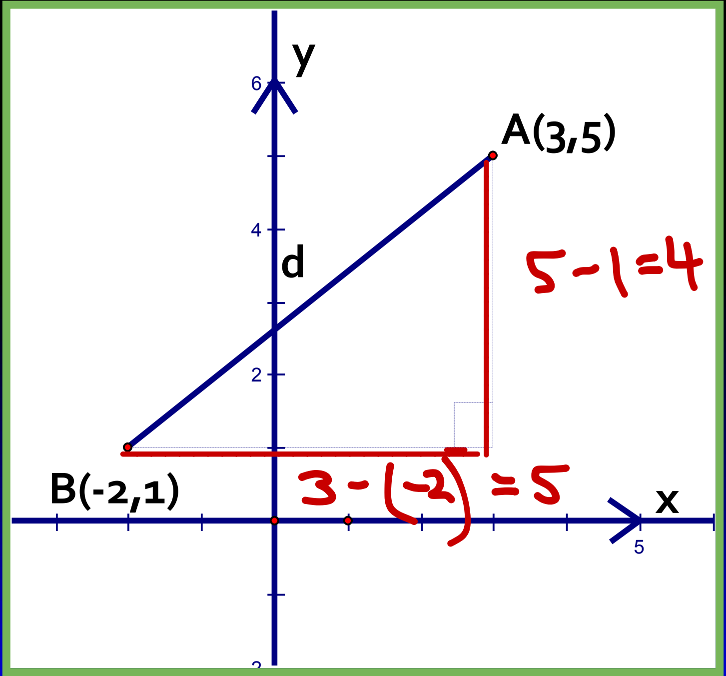
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Big Ideas (Continued)

- The Pythagorean theorem is used to develop the distance formula, by calculating the straight-line distance between two points.
- The distance between a point and a line is the shortest distance between them. It is measured on a perpendicular line from the point to the line.

Example #1

- Find the distance between the points A(3,5) and B(-2,1). Round to the nearest tenth.



$$d = \sqrt{(5-1)^2 + (3-(-2))^2}$$

$$d = \sqrt{(4)^2 + (5)^2}$$

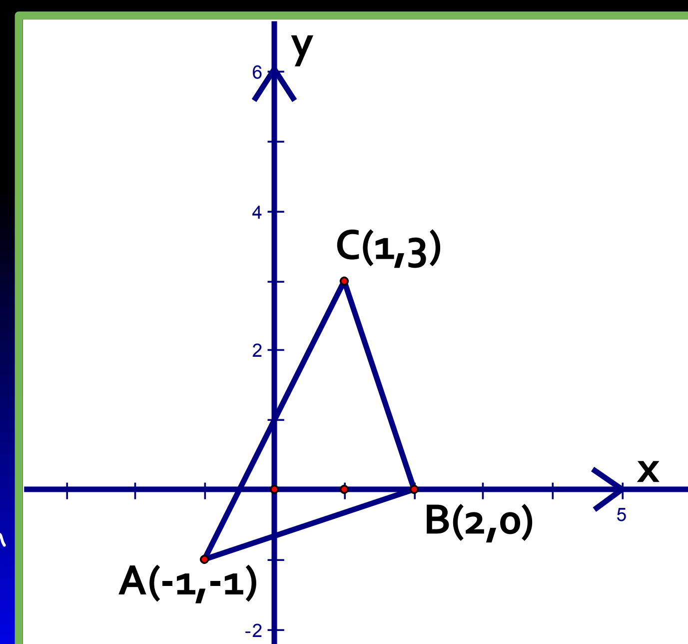
$$d = \sqrt{16+25}$$

$$d = \sqrt{41}$$

$$d \approx 6.4$$

Example #2

- A triangle has vertices at A(-1,-1), B(2,0), and C(1,3). Find the lengths and slopes of the sides of the triangle. What kind of triangle is it?



AC

Slope: $m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3-(-1)}{1-(-1)}$

$= \frac{4}{2}$

$m_{AC} = 2$

length: $d_{AC} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{2^2 + 4^2}$

$d_{AC} = \sqrt{4+16}$

$= \sqrt{20}$

$d_{AC} = 4.47$

Side CB

slope: $m_{CB} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0 - 3}{2 - 1}$

$= \frac{-3}{1}$

$m_{CB} = -3$

$d_{CB} = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

$= \sqrt{(-3)^2 + (1)^2}$

$= \sqrt{9 + 1}$

$= \sqrt{10}$

$d_{CB} = 3.16$

Side AB

slope: $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0 - (-1)}{2 - (-1)}$

$m_{AB} = \frac{1}{3}$

slopes are negative reciprocals
∴ Right triangle

$d_{AB} = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

$= \sqrt{(1)^2 + (3)^2}$

$= \sqrt{1 + 9}$

$= \sqrt{10}$

$d_{AB} = 3.16$

Isoscles

∴ $\triangle ABC$ is a right isosceles triangle.

Consolidation

- Know it! What was that formula again (without looking)?

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

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#6.

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$M(1, 1)$

$P(-3, -1)$

x coordinate of other point

$$1 = \frac{-3 + x_2}{2}$$

$2(1) = -3 + x_2$

$2 = -3 + x_2$

$2 + 3 = x_2$

$5 = x_2$

y coord. of other point

$$1 = \frac{-1 + y_2}{2}$$

$2(1) = -1 + y$

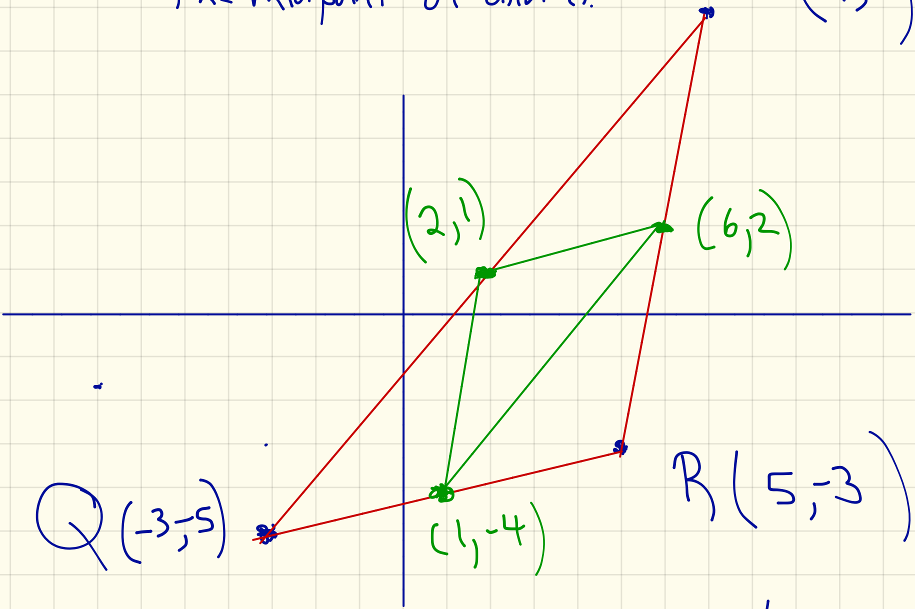
$2 = -1 + y$

$2 + 1 = y$

$3 = y$

∴ $(5, 3)$ is the other point.

midsegments – line segment from midpoint of 1 side to the midpoint of another.



$$M_{PQ} = \left(\frac{-3+7}{2}, \frac{-5+7}{2}\right)$$
$$= (2, 1)$$

$$M_{PR} = \left(\frac{7+5}{2}, \frac{7-3}{2}\right)$$
$$= (6, 2)$$

$$M_{QR} = \left(\frac{-3+5}{2}, \frac{-5-3}{2}\right)$$
$$= (1, -4)$$

Reinforcement

- Pages 86 - 87
  - #4abc, 5abc, 7, 10, 12a