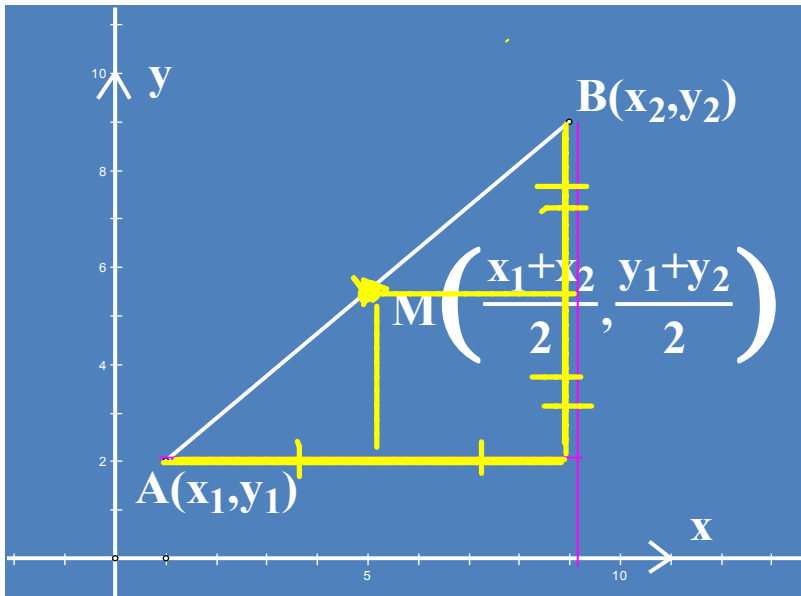# Midpoint of a Line Segment

# Learning Goals

- Develop and use the formula for the midpoint of a line segment.

# Big Ideas

- The coordinates of the midpoint of a line segment are the means (averages) of the coordinates of the end points.
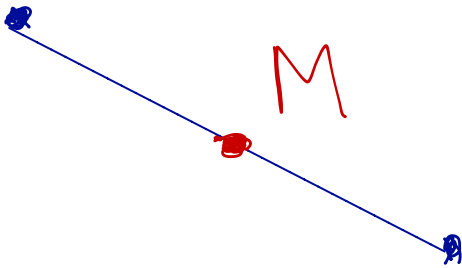


Given the line segment with end points $A(x_1, y_1)$ and $B(x_2, y_2)$, the midpoint is the point with the coordinates:

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

# Example #1

- Find the coordinates of the midpoint of the line segment with these end points, P(2,-4) and Q(-3,5).

$Q(-3,5)$

$M$

$P(2,-4)$

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left( \frac{-3 + 2}{2}, \frac{5 + (-4)}{2} \right)$$

$$M = \left( -\frac{1}{2}, \frac{1}{2} \right)$$

# Big Ideas (Continued)

- The coordinates of a midpoint can be used to determine an equation for a median in a triangle or the perpendicular bisector of a line segment.

HOW TO FIND THE EQUATION OF A MEDIAN:
1) Find the coordinates of the midpoint.
2) Use the midpoint and the opposite vertex to find the slope of the median.
3) Use either point to find the y-intercept.
4) Write the equation.

HOW TO FIND THE EQUATION OF A PERPENDICULAR BISECTOR:
1) Find the coordinates of the midpoint.
2) Use the vertices of the side to find the slope of the side.
3) Find the negative reciprocal, this is the slope of the perpendicular bisector.
4) Use the midpoint to find the y-intercept.
5) Write the equation.

# Example #2

- △STU has vertices S(-2,-3), T(9,4), and U(11,-4).

  a) Find the equation of the perpendicular bisector of side TU.

  b) Find the equation of the median from S.

  c) What do you notice?  What kind of triangle is △STU?

Equation ++++ and median are same. ∴ △STU is isosceles

# Solution for Example #2

① Perpendicular Bisector of TU

• Midpoint of TU

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{9+11}{2}, \frac{4+(-4)}{2}\right)$$

$$= \left(\frac{20}{2}, \frac{0}{2}\right)$$

$$M = (10, 0)$$

② Slope of TU

$$m = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{-4-4}{11-9}$$

$$= -\frac{8}{2}$$

$$m_{TU} = -4$$

③ Slope of perpendicular Bisector → $m = \frac{1}{4}$
is negative reciprocal of $m = -4$

$$-4 = \frac{-4}{1} \implies \frac{1}{4}$$

④ Equation of perpendicular bisector

$$m = \frac{1}{4}$$

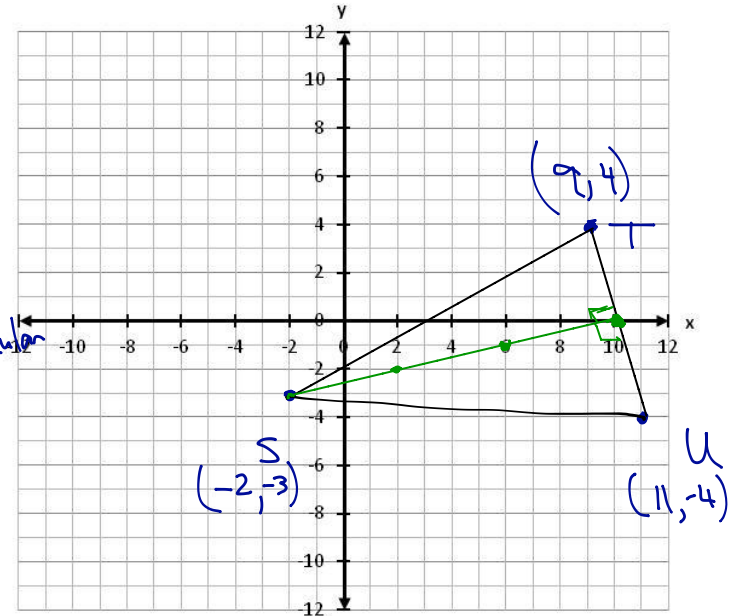$$M = (10, 0)$$
$$x, y$$

$$y = mx + b$$

$$0 = \frac{1}{4}(10) + b$$

$$0 = 2.5 + b$$

$$0 - 2.5 = 2.5 - 2.5 + b$$

$$b = -2.5$$

$$\therefore y = \frac{1}{4}x - 2.5$$
is equation of perpendicular bisector.

b) ① Midpoint of TU
$$(10, 0)$$

② Coordinates of S
$$(-2, -3)$$

③ Slope of median
$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{0-(-3)}{10-(-2)}$$
$$m = \frac{3}{13}$$

T (9, 4)

U (11, -4)

S (-2, -3)

③ con't.

  Slope of median  $m = \dfrac{3}{12}$

④  $m = \dfrac{1}{4}$

Use slope and 1 of the points

midpoint $(10, 0)$ or $S(-2, -3)$ to
find b.

$$y = mx + b$$
$$0 = \dfrac{1}{4}(10) + b$$
$$0 = 2.5 + b$$
$$0 - 2.5 = 2.5 - 2.5 + b$$
$$b = -2.5$$

∴ Equation of median from S
is $\boxed{y = \dfrac{1}{4}x - 2.5}$

# Reinforcement

- Pages 79 – 80
  - #4def, 5, 6, 11, 12, 13a