

Using Coordinates to Solve Problems

Quiz - Friday ~~→~~ 2.2 - 2.5

Chapter 2 Test - next Wednesday
Oct. 26.

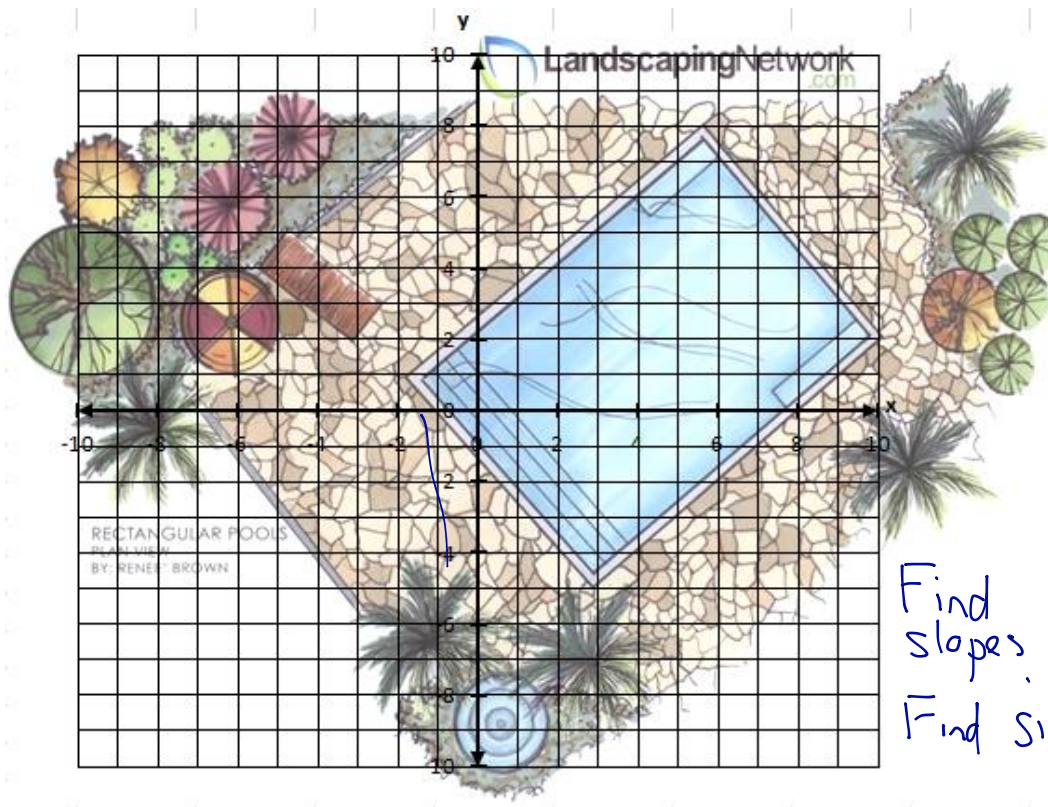
Learning Goal

- Use properties of lines and line segments to solve problems.

Minds on ...



SWIMMING POOLS Antonio is designing a swimming pool on a coordinate grid. Is it a rectangle? Explain.



Find slopes
Find side lengths

What should I do?

- Read the problem carefully.
- Highlight important information.
- Draw a picture.
- Determine what you need to find.
- Make a plan.
- Execute your plan, keeping it organized.
- State your solution.
- Check that your answer is reasonable.

G Given	R Required
A Analysis	S Steps / Solution
P	

Example #1

- On the design plan for a garden, a straight path runs from $(-25, 20)$ to $(40, 36)$. A lamp is going to be placed halfway along the path. Determine the coordinates for the lamp.

• Find the midpoint

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$M = \left(\frac{-25 + 40}{2}, \frac{20 + 36}{2} \right)$$

$$= \left(\frac{15}{2}, \frac{56}{2} \right)$$

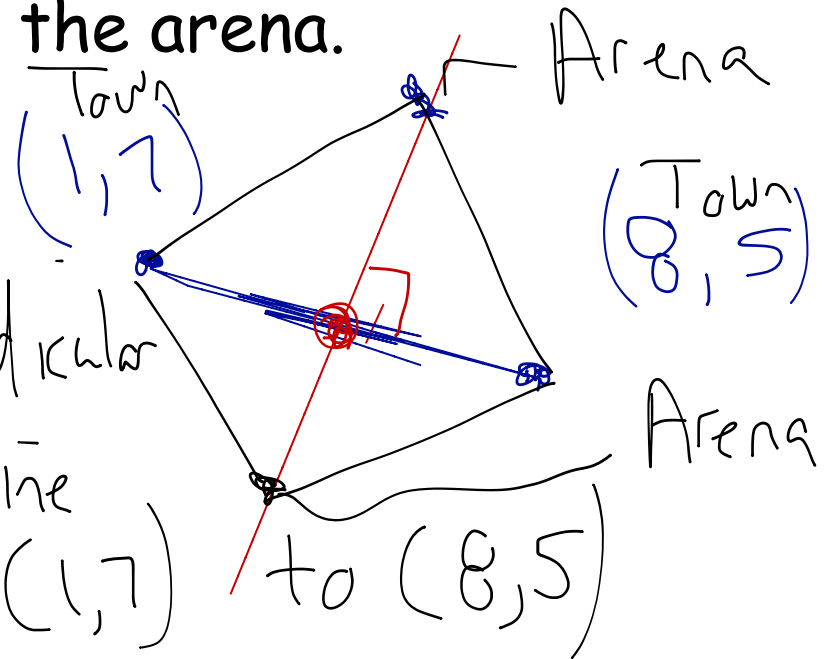
$$M = (7.5, 28)$$

∴ The lamp should be placed at $(7.5, 28)$

Example #2

- A hockey arena is going to be built to serve two rural towns. On a plan of the area, the towns are located at $(1, 7)$ and $(8, 5)$. If the arena needs to be the same distance from both towns, determine an equation to describe the possible locations for the arena.

We need the equation of the line that is the perpendicular bisector of the line that goes from $(1, 7)$



Plan: ① Determine midpoint of $(1, 7)$ and $(8, 5)$

② Find slope of line segment from $(1, 7)$ to $(8, 5)$

③ Slope of the perpendicular bisector is the negative reciprocal from ②

④ Use $y = mx + b$ with midpoint from ① and slope from ③ to figure out b

⑤ State the equation of the line

Execute Plan

① Midpoint $(1, 7)$ to $(8, 5)$ ② Slope

$$M = \left(\frac{1+8}{2}, \frac{7+5}{2} \right)$$

$$M = (4.5, 6)$$

$$\begin{matrix} x_1, y_1 & x_2, y_2 \\ (1, 7) & (8, 5) \end{matrix}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5 - 7}{8 - 1}$$

③ Negative Reciprocal of slope in ②

$$m = 7/2$$

$$m = -2/7$$

$$\textcircled{4} \quad M = \begin{pmatrix} x & y \\ 4.5 & 6 \end{pmatrix} \quad \text{Solve for } b$$

$$m = \frac{7}{2}$$

$$y = mx + b$$

$$b = \frac{7}{2}(4.5) + b$$

$$6 = \frac{7 \times 4.5}{2} + b$$

$$6 = \frac{31.5}{2} + b$$

$$6 = 15.75 + b$$

$$6 - 15.75 = b$$

$$-9.75 = b$$

$$\textcircled{5} \quad y = \frac{7}{2}x - 9.75$$

$$y = 3.5x - 9.75$$

\therefore The arena should be built
somewhere on the line $y = 3.5x - 9.75$

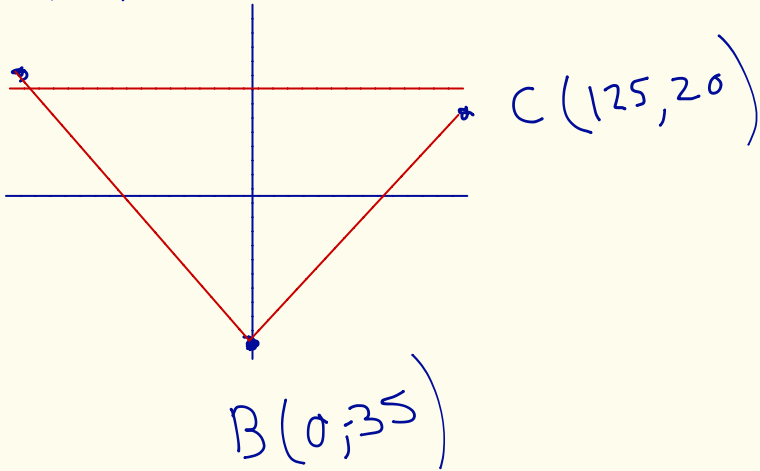
Example #3

- A new lookout tower is going to be built so that it is the same distance from three ranger stations. If the stations are at $A(-90, 28)$, $B(0, -35)$ and $C(125, 20)$ on a grid, determine the coordinates of the point where the new tower should be built.

We have to find the circumcentre
of $\triangle ABC$

↓
point where perpendicular
bisectors meet.

$$A(-90, 28)$$



$$B(0, 35)$$

$$C(125, 20)$$

- Plan
- ① Det. midpoint of each side
 - ② Det slope of each side
 - ③ slope perpendicular bisector is negative reciprocal of slope in ②
 - ④ Det. b for each perpendicular bisector using midpoint from ① and slope from ③
 - ⑤ Write equation for each perp. bisector.
 - ⑥ Determine P.O.I. for the perpendicular bisector.

Execute Plan

$$\textcircled{1} M_{AC} = (17.5, 24)$$

$$M_{AB} = (-45, 35)$$

$$M_{BC} = (62.5, -7.5)$$

$$\textcircled{2} m_{AC} = -8/215$$

$$m_{AB} = -63/90$$

$$m_{BC} = 55/125$$

$$\textcircled{3} m = 215/8$$

$$= -7/10$$

$$= 11/25$$

$$m = 10/7$$

$$m = -25/11$$

④ b
from AC

b
from AB

b
from BC

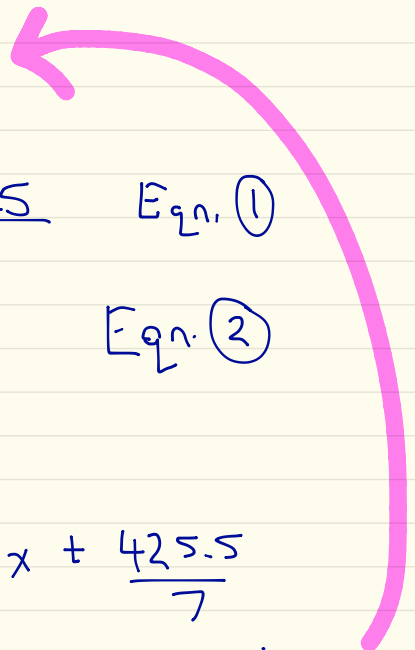
$$b = \frac{-3570.5}{8}$$

$$b = \frac{425.5}{7}$$

$$b = \frac{1480}{11}$$

⑤

$$y = \frac{215}{8}x - \frac{3570.5}{8} \qquad y = \frac{10}{7}x + \frac{425.5}{7} \qquad y = \frac{-25}{11}x + \frac{1480}{11}$$

$$8 \times 7 = 56$$


⑥ $y = \frac{215x}{8} - \frac{3570.5}{8}$ Eqn. (1)

$$y = \frac{10}{7}x + \frac{425.5}{7}$$
 Eqn. (2)

Sub (1) into (2)

$$\frac{215x}{8} - \frac{3570.5}{8} = \frac{10}{7}x + \frac{425.5}{7}$$

multiply both sides by 56 to get rid of fraction

$$7(215x) - 7(3570.5) = 8(10x) + 8(425.5)$$

$$1505x - 24993.5 = 80x + 3404$$

$$1505x - 80x = 3404 + 24993.5$$

$$1425x = 28397.5$$

$$x = 19.9$$

$$x \approx 20$$

② was

$$y = \frac{10}{7}x + \frac{425.5}{7}$$

$$y = \frac{10(20)}{7} + \frac{425.5}{7}$$

$$y = \frac{200}{7} + \frac{425.5}{7}$$

$$y = \frac{625.5}{7}$$

$$y = 89.3$$

$$y \approx 89$$

∴ The tower should be built at (20, 89). What do you notice?

Example #4

- A power line is going to be laid from $A (-22, 15)$ to $B (7, 33)$ to $C (10, 18)$ to $D (-1, 4)$. If the units are metres, what length will the power line be?

Consolidation

- What are the tools in your mathematical tool belt?



Reinforcement

- Pages 120 - 121
 - #8, 9, 12, 14, 16, & 17