

Factoring Quadratics: $ax^2 + bx + c$



Learning Goal

- Factor quadratic expressions of the form $ax^2 + bx + c$, where $a \neq 1$.



Minds on ...

- Remember expanding?

$$(2x - 3)(4x + 5)$$

$$= 8x^2 + 10x - 12x - 15$$

$$= 8x^2 - 2x - 15$$

Look for patterns!



Try something more algebraic...

- Decomposition
 - Factor $3x^2 + 11x + 6$

$$\begin{array}{l}
 3x^2 + 2x + 9x + 6 \\
 \downarrow \quad \quad \downarrow \\
 \text{Factor out } x \quad \text{Factor out } 3 \\
 \textcircled{7}(3x+2) + \textcircled{3}(3x+2) \\
 = (x+3)(3x+2)
 \end{array}$$

$$a = 3$$

$$b = 11$$

$$c = 6$$

$$(a)(c) = (3)(6) = 18$$

$$P = 18$$

$$S = 11$$

$$9 \times 2 = 18$$

$$9 + 2 = 11$$

STEPS:

1. Find the product of a and c .
2. Use the product of a and c and the sum b to find the magic numbers.
3. Rewrite the x -term as the sum of the magic numbers.
4. Factor by parts.
5. Common factor.



$$3x^2 + 11x + 6$$

$$3 \times 6 = 18$$

$$\frac{2}{3} \times \frac{9}{3} = \frac{18}{9} = 2$$

$$+ 11$$

$$\frac{1}{2} \times \frac{18}{9} = \frac{18}{18} = 1$$

lowest terms

$$(3x+2)(x+3)$$

$$\frac{2}{3} \times \frac{3}{1} = 2$$

What about guessing?

- Trial and error

- Factor $3x^2 + 11x + 6$

Trial #1

	3, 1	1, 6	
$3x$	\searrow	\nearrow	$1 = 1x$
$1x$	\nearrow	\searrow	$6 = 18x$
			$\frac{18x}{19x}$

X We need $11x$

$a = 3$ $c = 6$

3, 1 1, 6
3, 2

Trial #2

	3, 1	6, 1	
$3x$	\searrow	\nearrow	$6 = 6x$
$1x$	\nearrow	\searrow	$1 = 3x$
			$\frac{3x}{9x}$

X we need $11x$

Trial #3

	3, 1	2, 3	
$3x$	\searrow	\nearrow	$2 = 2x$
$1x$	\nearrow	\searrow	$3 = 9x$
			$\frac{9x}{11x}$

😊

$\therefore (3x + 2)(x + 3)$

STEPS:

1. Find all the factors of a.
2. Find all the factors of c.
3. Use a chart to cross-multiply the factors.
4. Find the sum of these products.
5. Check this sum with the sum b.
6. When you find a match, write these factors in the brackets.



Big Ideas

- You can use the following strategies to factor:
 - Decomposition
 - Trial and error
 - Magic X



Examples

- Factor the following using a method of your choice.

$$4x^2 - 8x - 5$$

$$a=4 \quad \begin{array}{l} / \quad \backslash \\ 2x \quad -10x \end{array}$$

$$c=-5$$

$$(a)(c) = (4)(-5) = -20$$

$$p: -20$$

$$s: -8$$

$$= 4x^2 + 2x - 10x - 5$$

Factor out 2x Factor out -5

$$(2x)(2x+1) - 5(2x+1)$$

$$= (2x-5)(2x+1) \quad \checkmark$$

$$6x^2 + 7x - 3$$

$$a=6 \quad \begin{array}{l} / \quad \backslash \\ 9x \quad -2x \end{array}$$

$$c=-3$$

$$(a)(c) = (6)(-3) = -18$$

$$p: -18$$

$$s: +7$$

$$\therefore 9x-2$$

$$6x^2 + 9x - 2x - 3$$

$$(3x)(2x+3) - 1(2x+3)$$

$$= (3x-1)(2x+3)$$



Consolidation

- Think, pair, and share
 - Take a minute to talk to your elbow partner about which method you think you will use to factor these more difficult trinomials and why that is your choice.



Reinforcement

- Read page 221 (examples 3 and 4) and page 222 Key Ideas and Need to Know
- Pages 223 – 224
- #3 – 7, 9, 15

