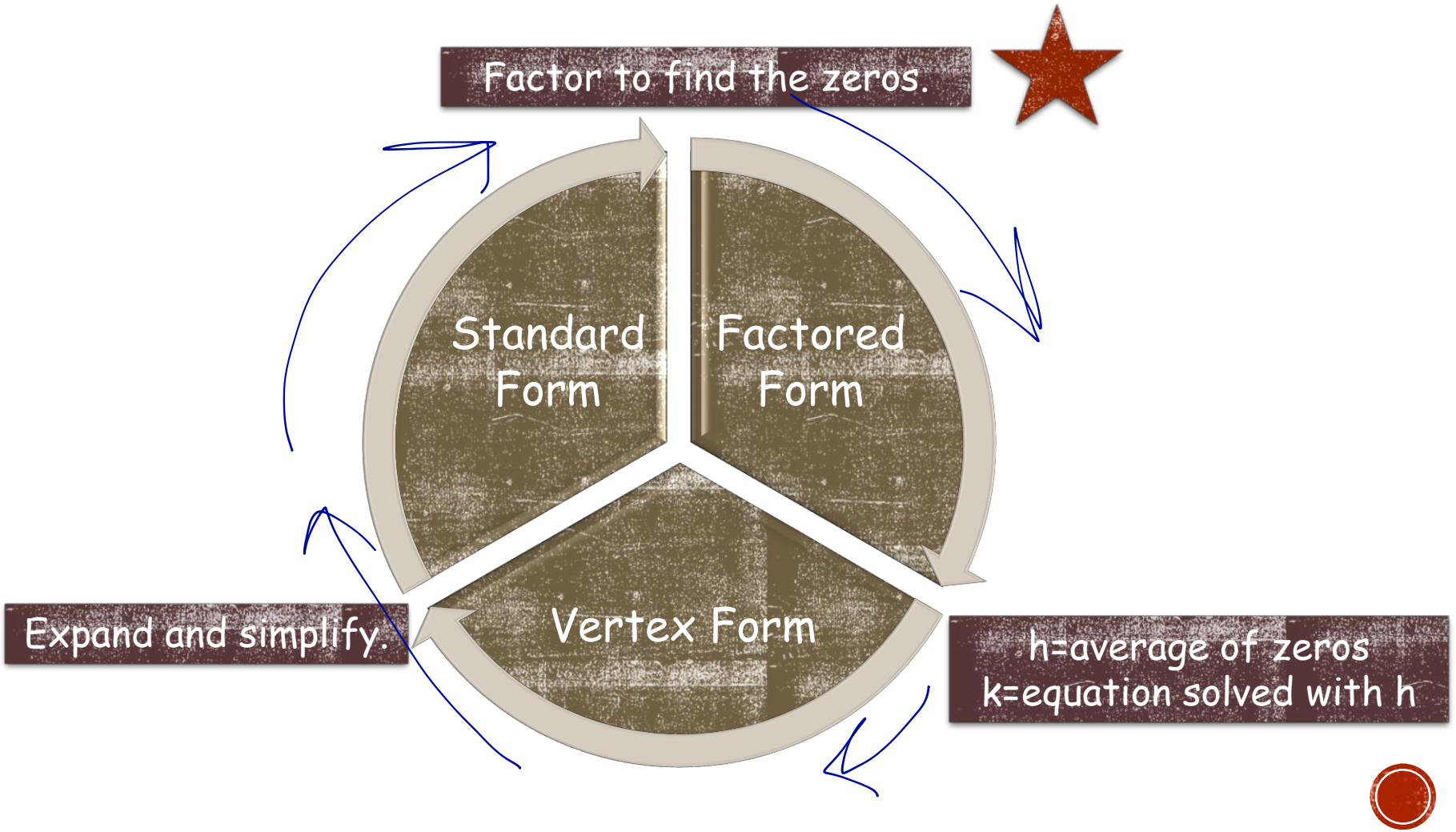
SOLVING PROBLEMS USING QUADRATIC RELATIONS

LEARNING GOAL

- Model and solve problems using the vertex form of a quadratic relation.

BIG IDEAS


Factor to find the zeros.
Standard Form
Factored Form
Vertex Form
Expand and simplify.
h=average of zeros
k=equation solved with h

EXAMPLE

- Graph the quadratic relation

  $y = -3x^2 + 12x + 15$ by finding the x-intercepts and the vertex

$y = -3(x^2 - 4x - 5)$ $\quad P: -5$, $S: -4$

$y = -3(x - 5)(x + 1)$

Zeros: $x - 5 = 0$ $\quad x + 1 = 0$

$x = 5$ $\quad x = -1$

A.O.S. $x = \frac{5 + (-1)}{2}$

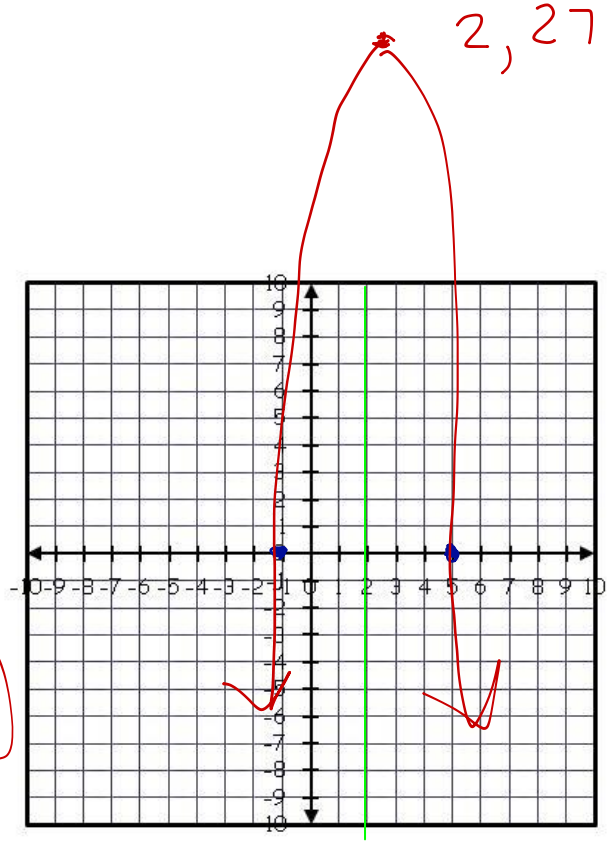
$x = \frac{4}{2}$

$x = 2$

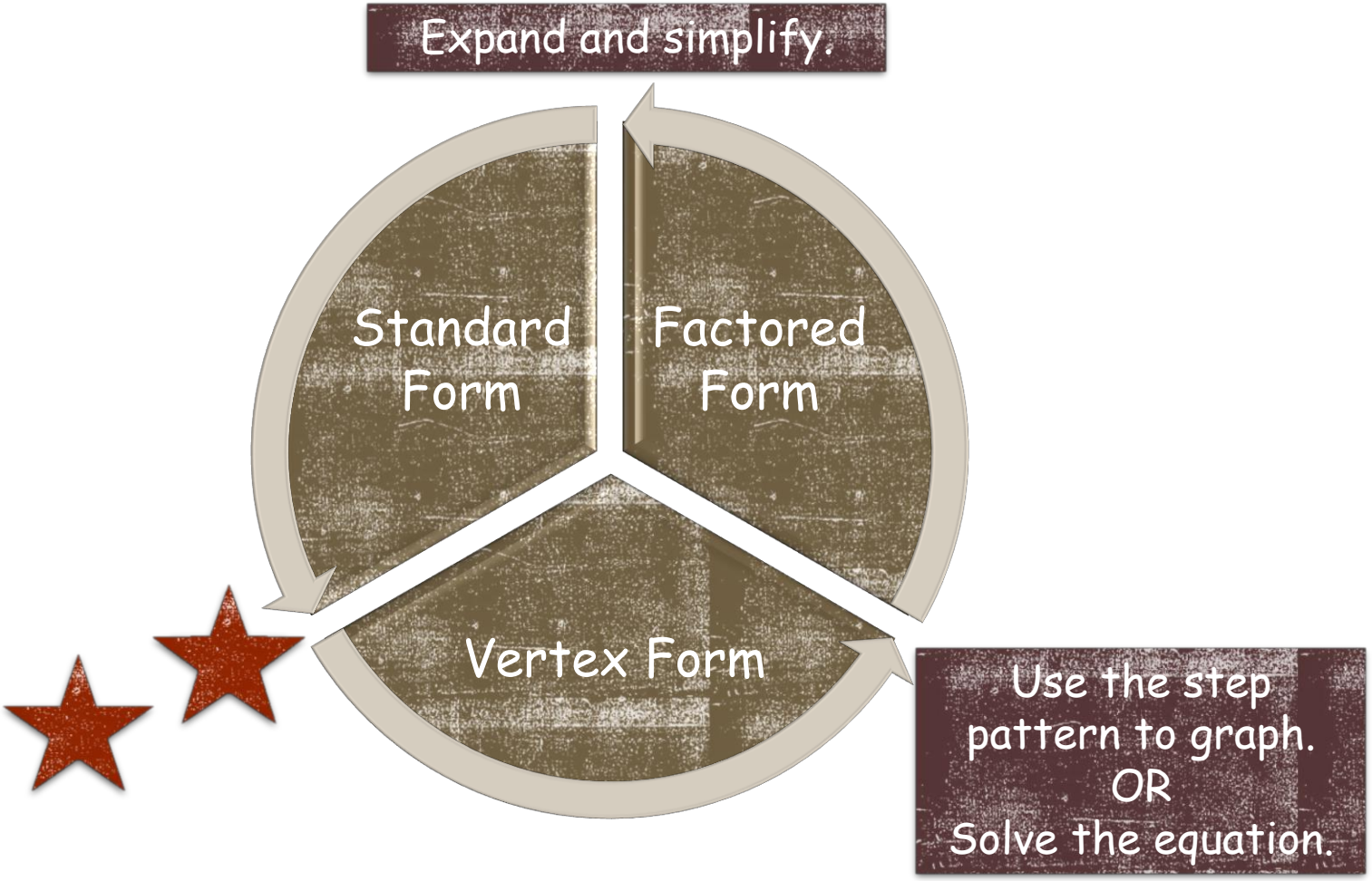
$y = -3(2 - 5)(2 + 1)$

$y = -3(-3)(3)$

$y = 27$



MORE BIG IDEAS



ANOTHER EXAMPLE

- Write the quadratic relation $y = (x - 2)^2 - 9$ in factored and standard forms.

① Expand & Simplify (Std form)

② Factor (Factored Form)

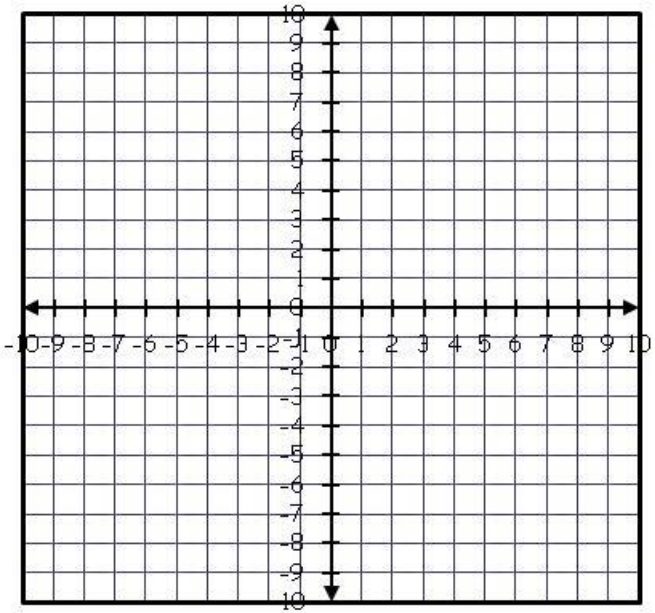
$y = (x-2)^2 - 9$

$y = (x-2)(x-2) - 9$

$y = x^2 - 2x - 2x + 4 - 9$

① $y = x^2 - 4x - 5$

② $y = (x-5)(x+1)$



CONSOLIDATION

- To go from vertex to factored form without graphing, you could also try first rewriting in standard form and then factoring this.

CONNECTING STANDARD AND VERTEX FORMS

LEARNING GOAL

- Sketch or graph a quadratic relation with an equation of the form $y = ax^2 + bx + c$ using symmetry.

MINDS ON … (CONT)

- Now let's put some math into it!
  - (a)(b) = 0

- Okay, now some x's!
  - (x – 3)(x + 4) = 0

$x - 3 = 0$ $\quad x = 3$ OR $x + 4 = 0$ $\quad x = -4$

- You just solved a quadratic equation!

MORE MINDS ON …

- Can you find a pair of symmetric points?
- Why are they symmetric?

(2, 8)

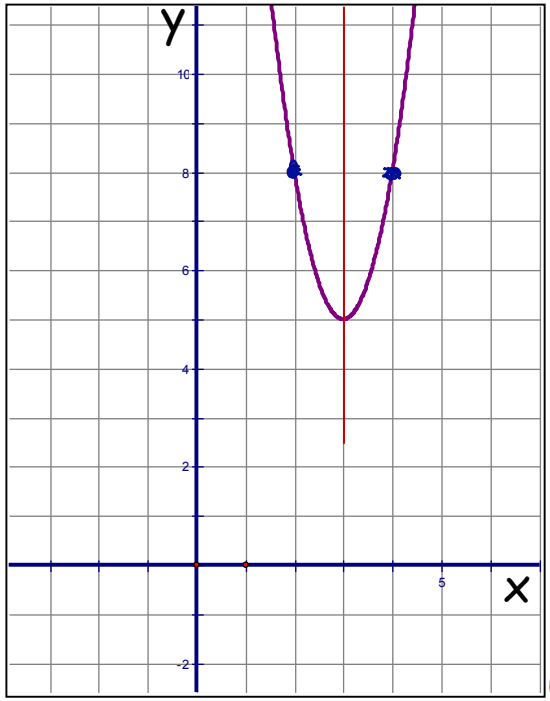
(4, 8)

A.O.S.

$x = \frac{2+4}{2}$

$= \frac{6}{2}$

$x = 3$



MORE MINDS ON … (CONT)

- Not all equations have zeros, but they do all have a vertex.
- We can therefore write all equations in vertex form even if we cannot write them in factored form.
- Instead of using zeros to find the vertex, we use symmetric points and solve a quadratic equation.

BIG IDEAS

- You can use a technique called partial factoring to find the vertex form of a quadratic relation given standard form:
  - Common factor the x^2- and x-terms.
  - Let y equal the value of the constant term.
  - Solve the resulting quadratic equation.
  - This gives two symmetric points on the curve.
  - Use the symmetric points to find the vertex.

EXAMPLE 1

- Find the vertex form of $y = x^2 + 2x + 5$.

$y = x(x+2) + 5$

Let $y = 5$

$5 = x(x+2) + 5$

$5 - 5 = x(x+2) + 5 - 5$

$0 = x(x+2)$

$x + 2 = 0$

$x = -2$

$x = 0$

AOS $x = \frac{-2+0}{2}$

$x = -1$

2 points on the parabola are $(0, 5)$ and $(-2, 5)$

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#1, 4, 7b

EXAMPLE 2

- Find the vertex form of $y = -3x^2 + 42x - 147$.

Vertex Form

$y = a(x-h)^2 + k$

$y = -3(x-7)^2 + 0$

$y = -3(x-7)^2$

$y = -3x(x-14) - 147$

Let $y = -147$

$-147 = -3x(x-14) - 147$

$-147 + 147 = -3x(x-14) - 147 + 147$

$0 = -3x(x-14)$

$-3x = 0$ $\quad x - 14 = 0$

$x = 0$ $\quad x = 14$

$\therefore$ $(0, -147)$ and $(14, -147)$ are 2 pts on the parabola.

$\therefore$ A.O.S.

$x = \frac{0+14}{2}$

$x = 7$

y coordinate of vertex

$y = -3(7)^2 + 42(7) - 147$

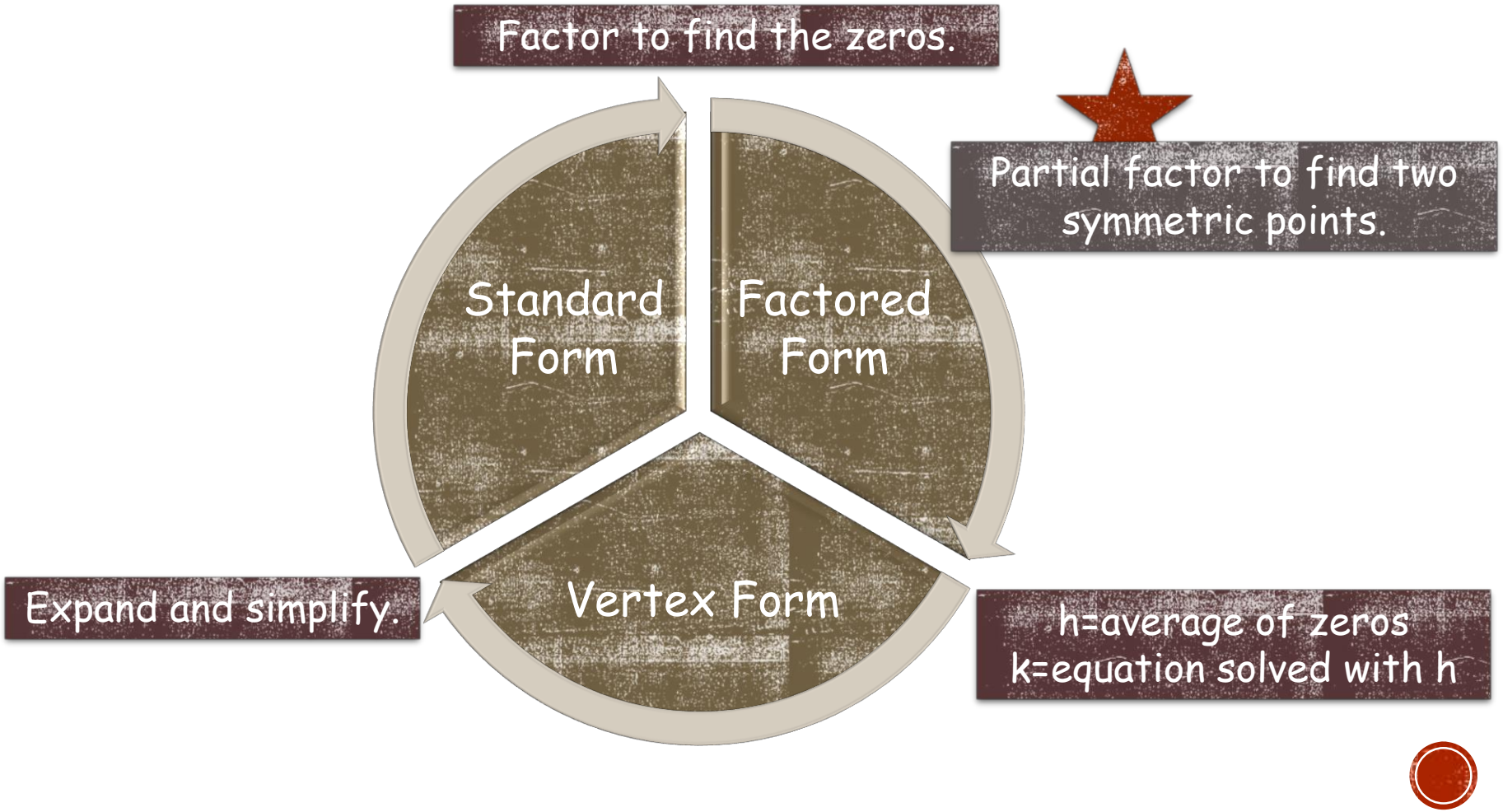
$y = -3(49) + 294 - 147$

$y = -147 + 294 - 147$

$y = 0$

Vertex $(7, 0)$

CONSOLIDATION



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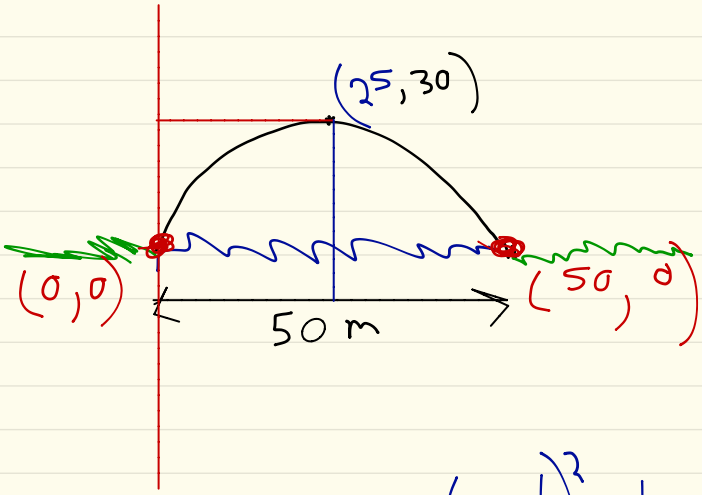

Vertex Form: $y = a(x-h)^2 + k$

$y = a(x-25)^2 + 30$

$0 = a(0-25)^2 + 30$

A point on the parabola is $(x, y) = (0, 0)$

$0 = a(-25)^2 + 30$

Solve for a:

$-30 = 625a$

$a = \frac{-30}{625}$

$a = -0.048$

$y = -0.048(x-25)^2 + 30$

8 m from the axis of symmetry.

A.O.S. $x = 25$

$25 - 8 = 17$ m

$25 + 8 = 33$ m

$y = -0.048(x-25)^2 + 30$

$y = -0.048(33-25)^2 + 30$

$y = -0.048(8)^2 + 30$

$y = -0.048(64) + 30$

$y = -3.072 + 30$

$y = 26.928$

∴ The sailboat will fit!

REINFORCEMENT

- Pages 293 – 295
  - #5a, 6a, 7, 9, 10, 11, 16, 17*

REINFORCEMENT

- Pages 301 – 302
  - #7, 8, 9, 12, 13, 14