COMPLETING THE SQUARE

Learning Goals

- Recognize the relationship between the coefficients and constants of perfect-square trinomials.
- Write the equation of a parabola in vertex form by completing the square.

Minds on...

- Infomercial for today's lesson...
  - Are you annoyed with all the work you have to do with partial factoring?
  - Do you sometimes wonder if there is an easier way to go from vertex form to standard form without all the hassle?
  - Well, there is!
  - After you learn to complete the square you'll be able to finish the question in just 5 easy steps!

Big Idea

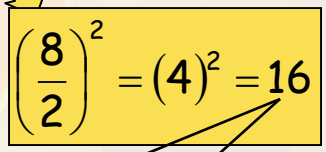
- Completing the square is a technique which allows you to rewrite standard form into vertex form without finding any symmetric points.

Steps to Complete the Square:

1. Find "a". *(If necessary)*
   a) Common factor only the coefficients of the quadratic and linear terms.
2. Add zero.
   a) Determine what constant would create a perfect square by dividing the linear coefficient by 2 and squaring it.
   b) Add and subtract this number at the same time.
3. Create the perfect square. *(If necessary)*
   a) If there was a value for "a", multiply the negative constant by this to remove it from the bracket.
4. Factor the perfect square.
5. Simplify the constants.

Example

1. Rewrite $y = x^2 + 8x + 13$ in vertex form by completing the square.



Add zero

$y = x^2 + 8x + 13$

$y = x^2 + 8x + 16 - 16 + 13$

FACTOR SIMPLIFY

$y = (x + 4)^2 - 3$

Example

2. Rewrite $y = x^2 - 18x + 100$ in vertex form by completing the square.

$y = x^2 - 18x + 100$

$\left(\frac{-18}{2}\right)^2 = (-9)^2 = 81$

$y = x^2 - 18x + 81 - 81 + 100$

$y = (x - 9)^2 + 19$

Vertex $(9, 19)$

Example

2. Rewrite $y = x^2 - 18x + 100$ in vertex form by completing the square.

$$\left(\frac{-18}{2}\right)^2 = (-9)^2 = 81$$

$$y = x^2 - 18x + 100$$

$$y = x^2 - 18x + 81 - 81 + 100$$

$$y = (x - 9)^2 + 19$$

Example

3. Rewrite $y = 2x^2 + 8x + 9$ in vertex form by completing the square.

$y = 2x^2 + 8x + 9$

$y = (2x^2 + 8x) + 9$

$= 2(x^2 + 4x) + 9$

$= 2(x^2 + 4x + 4 - 4) + 9$

$= 2(x^2 + 4x + 4) - 8 + 9$

$\left(\frac{4}{2}\right)^2 = 2^2$

$= 4$

$$y = 2(x^2 + 4x + 4) - 8 + 9$$

$$y = 2(x + 2)^2 + 1$$

Vertex $(-2, 1)$

Example

3. Rewrite $y = 2x^2 + 8x + 9$ in vertex form by completing the square.

$y = 2x^2 + 8x + 9$

$\left(\frac{4}{2}\right)^2 = (2)^2 = 4$

$y = 2(x^2 + 4x) + 9$

$y = 2(x^2 + 4x + 4 - 4) + 9$

$y = 2(x^2 + 4x + 4) - 8 + 9$

$y = 2(x + 2)^2 + 1$

Example

4. Find the vertex of the given relation, $y = 3x^2 - 30x + 79$, by completing the square.

$y = 3x^2 - 30x + 79$ $\qquad \left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$

$y = 3(x^2 - 10x) + 79$

$y = 3(x^2 - 10x + 25 - 25) + 79$

$y = 3(x^2 - 10x + 25) - 75 + 79$

$y = 3(x - 5)^2 + 4$

$\therefore$ the vertex is (5, 4).

Example

5. Find the vertex of the given relation, $y = -6x^2 + 12x - 5$, by completing the square.

$y = -6x^2 + 12x - 5$

$y = (-6x^2 + 12x) - 5$

$\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$

$y = -6(x^2 - 2x) - 5$

$y = -6(x^2 - 2x + 1 - 1) - 5$

$y = -6(x^2 - 2x + 1) + 6 - 5$

$y = -6(x - 1)^2 + 1$

Vertex $(1, 1)$

Example

5. Find the vertex of the given relation, $y = -6x^2 + 12x - 5$, by completing the square.

$y = -6x^2 + 12x - 5$

$\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$

$y = -6(x^2 - 2x) - 5$

$y = -6(x^2 - 2x + 1 - 1) - 5$

$y = -6(x^2 - 2x + 1) + 6 - 5$

$y = -6(x - 1)^2 + 1$

$\therefore$ the vertex is (1, 1).

Consolidation

- What if there is a decimal? For example, complete the square of $y = 0.5x^2 - 4x - 8$ to express the relation in vertex form.

$y = 0.5x^2 - 4x - 8$

$y = (0.5x^2 - 4x) - 8$

$y = 0.5(x^2 - 8x) - 8$

$y = 0.5(x^2 - 8x + 16 - 16) - 8$

$y = 0.5(x^2 - 8x + 16) - 8 - 8$

$y = 0.5(x - 4)^2 - 16$

$\left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

Reinforcement

- Pages 331 – 332 #5, 8, 9, 10, 11