Tree Diagrams
Key Ideas

• A tree diagram is used to count all the possible outcomes of a sequence of experiments or simple events where each event can occur in a finite number of ways.
Example

1. Matt and Darren are to play a tennis tournament. The first person to win 2 games in a row or who wins a total of 3 games wins the tournament. Find the number of ways the tournament can play out.
Solution

5 \times 2 = 10
Example

2. 1600 ants start climbing a tree, and at each branching, ¼ go left and ¾ go right, according to the diagram.
Solution

a) How many ants end up on branch C? 75
b) What is the probability of ending up at branch C? 
\[ P(C) = \frac{75}{1600} = \frac{15}{320} = \frac{3}{64} \]

c) What is \( P(G) \)? 
\[ 225 = \frac{45}{1600} = \frac{9}{320} = \frac{9}{64} \]
d) How many ants reach G? 225
Rule

• In general, the probability of events that follow sequentially on a tree diagram is found by multiplying the branch probabilities.
Example

3. A drawer contains 4 red and 3 blue socks.
   a) Draw three socks randomly and replace each one before drawing another (draw with replacement). Find the probability of drawing:
      i. Exactly 2 red socks.
      ii. At least 2 red socks.
   b) Draw three socks randomly without replacement. Find the probability of drawing:
      i. Exactly 2 red socks.
      ii. At least 2 red socks.
Solution 3 (a)

4 red
3 blue

\[ \frac{4 \times 1 \times 2}{1 \times 2 \times 3} = \frac{4}{6} = \frac{2}{3} \]

\[ \frac{3 \times 2 \times 1}{1 \times 2 \times 3} = 1 \]

\[ \frac{2 \times 1 \times 3}{1 \times 2 \times 3} = \frac{3}{3} = 1 \]

\[ \frac{1 \times 3 \times 2}{1 \times 2 \times 3} = 1 \]

\[ \left( \frac{48}{343} \right)^3 \]
At least 2 means

\[ \frac{2 \text{ or } 3}{P(2 \text{ Red}) = \frac{144}{343}} \]

\[ P(\text{RRR}) = \left( \frac{4}{7} \right) \left( \frac{4}{7} \right) \left( \frac{4}{7} \right) = \frac{64}{343} \]

\[ P(\text{at least 2 Red}) = \frac{144 + 64}{343} = \frac{208}{343} \]
Solution 3 (b)

\[ \text{RRB} = \left( \frac{4}{7} \right) \left( \frac{3}{6} \right) \left( \frac{3}{5} \right) = \frac{36}{210} \]

\[ \text{RBR} = \left( \frac{4}{7} \right) \left( \frac{3}{6} \right) \left( \frac{3}{5} \right) = \frac{36}{210} \]

\[ \text{BRR} = \left( \frac{3}{7} \right) \left( \frac{4}{6} \right) \left( \frac{3}{5} \right) = \frac{36}{210} \]

\[ \text{P( 0 of 2 Red) } = \frac{108}{210} \]

\[ \text{P( RRRR) } = \left( \frac{4}{5} \right) \left( \frac{3}{6} \right) \left( \frac{4}{5} \right) = \frac{24}{210} \]
Homework

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Probabilities

Using Counting Techniques
Key Ideas

- In calculating probabilities, $P(A)$, we usually:
  - List the outcome set, $S$;
  - List the event set, $A$; and
  - Count the number of elements in each list, that is, $n(S)$ and $n(A)$.

- There are useful techniques to aid us in accomplishing this:
  - Tree diagrams
  - Permutations
  - Combinations

\[ P(A) = \frac{n(A)}{n(S)} \]
1. A box contains 3 defective light bulbs and 7 good bulbs.
   a) Construct a tree diagram illustrating the various possibilities relating to 3 consecutive bulbs being drawn at random from the box without replacement.
   b) Find the probability that at least one good bulb is drawn from the box.
Tree Diagram / Indirect Method (cont)

\[
P(\text{at least 1 good bulb}) = P(\text{exactly 3 good}) + P(\text{exactly 2 good}) + P(\text{exactly 1 good})
\]

<table>
<thead>
<tr>
<th>Event</th>
<th>Formula</th>
<th>Calculation</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exactly 3 good</td>
<td>( P(\text{GGG}) )</td>
<td>( \frac{7 \times 6 \times 5}{10 \times 9 \times 8} )</td>
<td>0.29</td>
</tr>
<tr>
<td>Exactly 2 good</td>
<td>( P(\text{GGB}) + P(\text{GBG}) + P(\text{BGG}) )</td>
<td>( \left( \frac{7 \times 6 \times 3}{10 \times 9 \times 8} \right) + \left( \frac{7 \times 3 \times 6}{10 \times 9 \times 8} \right) + \left( \frac{3 \times 7 \times 6}{10 \times 9 \times 8} \right) )</td>
<td>0.525</td>
</tr>
<tr>
<td>Exactly 1 good</td>
<td>( P(\text{GBB}) + P(\text{BGB}) + P(\text{BBG}) )</td>
<td>( \left( \frac{7 \times 3 \times 2}{10 \times 9 \times 8} \right) + \left( \frac{3 \times 7 \times 2}{10 \times 9 \times 8} \right) + \left( \frac{3 \times 2 \times 7}{10 \times 9 \times 8} \right) )</td>
<td>0.175</td>
</tr>
</tbody>
</table>

\[
P(\text{at least 1 good bulb}) = 0.29 + 0.525 + 0.175 = 0.99
\]

It would be easier to calculate \( P(\text{at least 1 good bulb}) \) using an indirect method:

\[
P(\text{at least 1 good bulb}) = 1 - P(\text{all defective bulbs})
\]

\[
= 1 - \left( \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} \right)
\]

\[
= 1 - 0.01
\]

\[
= 0.99
\]
Observations

• Tree diagrams often become quite cumbersome and complicated.

• Therefore, we may bypass listing the outcome and event sets and just calculate the size of the sets directly, that is, \( n(S) \) and \( n(A) \), by using counting principles based on permutations and combinations.
Permutations

2. Nine horses are entered in a race. In an attempt to predict the finish of the race, three horses are selected by lot to finish first, second, and third. What is the probability that the choice is correct?
Permutations (cont)

• The outcome set consists of the ways that three horses chosen from nine can place first, second, and third.

\[
\frac{n(S)}{P_3} = 9^3 = 504
\]

• The event set contains exactly one element, the outcome where the order is correct.

\[
n(A) = 1 \quad P(A) = \frac{1}{504}
\]

\[
\therefore \text{the probability that the choice of the finish is correct is } \frac{1}{504}.
\]
3. A committee of five medical professionals is to be selected from ten doctors and eight nurses. What is the probability that there are exactly three doctors on the committee? (Assume each outcome is equally likely.)
Combinations (continued)

• The outcome set is to select 5 from a set of 18 (10 doctors, 8 nurses).
  \[ n(s) = \binom{18}{5} = 8568 \]

• The event set is to select 3 doctors from 10 doctors that leaves 2 nurses from 8 nurses.
  \[ n(A) = \binom{10}{3} \times \binom{8}{2} = 120 \times 28 = 3360 \]
  \[ P(A) = \frac{3360}{8568} = 0.4 \]

• The probability of selecting a committee with exactly three doctors is approximately 0.4 or 40%.
Homework

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